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TENSOR PRODUCT IN DETOUR RADIAL GRAPH

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Abstract

In this paper, the Tensor Product in Detour Radial graph DR(G) for some standard graphs are determined. Also we introduced b-Radial graph. The maximal energy and minimal energy are defined and they used to find the energy of Tensor Product in Detour Radial graph .

Index terms: Energy, Tensor Product, Radial graph, Detour Radial graph, b-eccentricity, b-radius.

1 INTRODUCTION

By a graph, we means finite simple and connected graph. For basic graph theoretical terminology we refer to Harary [7]. In a graph G , the detour distance D(u,v) between a pair of vertices u and v is the length of a longest path joining them. The Detour eccentricity $e_D(G)$ of a vertex u is the distance to a vertex farthest from u . The Detour radius $r_D(G)$ is the minimum detour eccentricity among the vertices of G and the detour diameter $d_D(G)$ is the maximum detour eccentricity among the vertices of G . A graph G for which $r_D(G)$ = $d_D(G)$ is called a self-centered graph. A vertex v is called a Detour eccentric vertex of u if D(u,v)= $e_D(G)$. A vertex v of G is called an detour eccentricity vertex of G if it is the eccentric vertex of some vertex of G . Let S be a subset of the vertex set of G such that e (u) D =i for all i u $\in S$.

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If v is an eccentric vertex of u and w is a neighbor of v, then $d(u,w) \le d(u,v)$. A vertex v may have this property, however, without being an eccentric vertex of u. The properties of eccentric vertices are studied in [10].

A vertex v is defined to be a boundary vertex u if $d(u,w) \le d(u,v)$ for all $w \in N(v)$. In [10] proved that the boundary set of any graph is geodestic, that is, every vertex in G lies on some shortest path joining two boundary vertices. The boundary vertices for a vertex may occur at different distance levels.

Let G be a connected graph. The b-eccentricity $e_b(u)$ of a vertex u is defined by $e_b(u) = \min\{d(u,v):$ $w \text{ is a boundary of } u\}$. The minimum b-eccentricity among the vertices of a graph G is b-radius $r_b(G)$ of G and the maximum b-eccentricity is its b-diameter $d_b(G)$.

Definition 1.1

Two vertices of a graph are said to be Detour Radial to each other if the detour distance between them is equal to the Detour Radius of the graph. A detour radial graph of a graph G denoted by DR(G) and it has the same vertex set as in G and two vertices

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are adjacent in $DR(G)$ if and only if they are detour radial in G.

In this paper, we introduced b-Radial graph $R_b(G)$ and Tensor Product on some standard Detour Radial graph $DR(G)$ are determined. The maximal and minimal energy are introduced to find the energy of $DR(G)$ are studied.

2 PRELIMINARIES

Theorem 2.1[6]

Let P_n be any path on n vertices. Then $DR(P_2) = P_2$, $DR(P_3) = P_3$ and

$$DR(P_n) = \begin{cases} \left(\frac{n}{2}\right)P_2 & , if\ \ n\ is\ even\ and\ n \geq 4 \\ P_3 \cup \left(\frac{n-3}{2}\right)P_2 & , if\ \ n\ is\ odd\ and\ n \geq 5 \end{cases}$$

Theorem 2.2[6]

Let C_n be any cycle on $n \geq 3$ vertices, then

$$r_D(C_n) = \begin{cases} \left(\frac{n}{2}\right) & , if\ \ n\ is\ even \\ \left(\frac{n+1}{2}\right) & , if\ \ n\ is\ odd \end{cases}$$

Theorem 2.3[6]

Let C_n be any cycle on $n \geq 3$ vertices, then

$$DR(C_n) = \begin{cases} \left(\frac{n}{2}\right)P_2 & , \ if\ \ n\ is\ even \\ \cong C_n & , \ if\ \ n\ is\ odd \end{cases}$$

3 MAIN RESULTS

3.1 Some Results on b-Radial Graph

Definition: 3.1.1

Two vertices of a graph are said to be b-Radial to each other if the distance between them is equal to the b-Radius of the graph. A b-Radial graph of a graph G denoted by $R_b(G)$ and it has the same vertex set as in G and two vertices are adjacent in $R_b(G)$ iff they are b-radial in G.

Theorem 3.1.2

Let P_n be any path on n vertices, then $r_b(P_n) = 1$

Proof.

Let $v\ (P_n)$={ $v_1, v_2, v_3........v_n$ }.

Let P_n be a connected graph. The b-eccentricity $e_b(u)$ of a vertex u is defined by

$e_b(u) = \min\{d(u,v):$
$w\ is\ a\ boundary\ of\ u\}$. ---------------- (1)

The minimum b-eccentricity among the vertices of a graph P_n is b-radius of P_n it is denoted as $r_b(P_n)$

i.e., $r_b(P_n) = \min\{b-eccentricity\ of\ \ P_n)$ -------------- (2)

Hence, by the equation (1) and (2)

$r_b\ (P_n)$= 1

Theorem 3.1.3

Let P_n be any path on n vertices. Then $R_b(P_n) = P_n$

Proof.

Let $v\ (P_n)$={ $v_1, v_2, v_3........v_n$ } , $r_b(P_n)$= 1

By the theorem 3.1.1 and definition,

$$R_b(P_n) = P_n$$

Theorem 3.1.4

Let C_n be any cycle on $n \geq 3$ vertices,

then $$r_b(C_n) = \begin{cases} \left(\frac{n}{2}\right) & , if \ \ n \ is \ even \\ \left(\frac{n-1}{2}\right) & , if \ \ n \ is \ odd \end{cases}$$

Proof. The result follows from Theorem 3.1.1

Theorem 3.1.5

Let C_n be any cycle on $n \geq 3$ vertices,

then $$R_b(C_n) = \begin{cases} \left(\frac{n}{2}\right) P_2 \ , & if \ \ n \ is \ even \\ \cong C_n & , if \ \ n \ is \ odd \end{cases}$$

Proof.

Let v (C_n)={ $v_1, v_2, v_3........v_n$ }

Case 1:

When n is even, then b-Radius C_n is $\frac{n}{2}$. A vertex and its b-eccentric vertex are b-Radial to each other.

i.e., $v_i \ and \ v_{\left(\frac{n+2i}{2}\right)}; i = 1,2,3,....\frac{n}{2}$ have the length of b-Radius.

Hence $R_b(C_n) = \left(\frac{n}{2}\right)$ disjoint copies of P_2

$$R_b(C_n) = \left(\frac{n}{2}\right) P_2$$

Case 2:

When n is odd,

Then b-Radius of C_n is $\left(\frac{n-1}{2}\right)$. $R_b(C_n)$ is the cycle with closed path $v_1 v_{r+1} v_{2r+1} v_r v_{2r} v_{r-1} ....v_{2r+2} \ v_1$, which is isomorphic to C_n.

i,e,. $R_b(C_n) \cong C_n$.

Hence,

$$R_b(C_n) = \begin{cases} \left(\frac{n}{2}\right) P_2 \ , & if \ \ n \ is \ even \\ \cong C_n & , if \ \ n \ is \ odd \end{cases}$$

3.2 Detour Radial in Tensor Product and Tensor Product in Detour Radial Graph

In this section, we take tensor product with P_2 and some standard graphs.

Theorem 3.2.1

Let P_n be a graph with n vertices, then

$$DR(P_2 \otimes P_n) = \begin{cases} n \ P_2 & , if \ \ n \ is \ even \ and \ \ n \geq 4 \\ 2P_3 \cup (n-3) P_2 & , if \ \ n \ is \ odd \ and \ \ n \geq 5 \end{cases}$$

Proof:

Let v (P_n)={ $v_1, v_2, v_3........v_n$ } .

Now, the Tensor product of path with two vertices and path with n vertices is given as two copies of path with n vertices.

By theorem 2.1, if n be even and odd vertices then the Detour Radial Tensor product of path with two vertices and path with n vertices is given as n copies of path with two vertices and two copies of path with three vertices union of (n-3) copies of path with two vertices.

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Theorem 3.2.2

Let C_n be any cycle on $n \geq 3$ vertices, then $DR(P_2 \otimes C_n) = n\,P_2$

Theorem 3.2.3

Let K_n be a complete graph with n vertices and P_2 be a path with 2 vertices, then

$DR(P_2 \otimes K_n) = K_{2n}$

Theorem 3.2.4

Let $K_{1,n}$ be a Star graph with n vertices and P_2 be a path with 2 vertices, then $DR(P_2 \otimes K_{1,n}) = K_{1,n}$

Theorem 3.2.5

Let P_n be a graph with n vertices, then

$$DR(P_2) \otimes DR(P_n) = \begin{cases} n\,P_2 & , n \text{ is even } \& \ n \geq 4 \\ 2P_3 \cup (n-3)P_2 & , n \text{ is odd } \& \ n \geq 5 \end{cases}$$

Theorem 3.2.6

Let C_n be any cycle on $n \geq 3$ vertices, then

$$DR(P_2) \otimes DR(C_n) = \begin{cases} 2^{\left(\frac{n}{2}\right)}\,P_2 & , if \ n \text{ is even and } n \geq 4 \\ C_{2n} & , if \ n \text{ is odd and } n \geq 3 \end{cases}$$

Theorem 3.2.7

Let K_n be a complete graph with n vertices and P_2 be a path with 2 vertices, then $DR(P_2) \otimes DR(K_n) = S_{2n}^{0}$

Theorem 3.2.8

Let $K_{1,n}$ be a Star graph with n vertices and P_2 be a path with 2 vertices, then $DR(P_2) \otimes DR(K_{1,n}) = K_{1,n}$

Remark:

- Let P_n be any graph with n vertices then, Detour radial of tensor product with P_2 and P_n is equal to Tensor Product with Detour radial of P_2 and Detour radial of P_n.

  i.e., $DR(P_2 \otimes P_n) = DR(P_2) \otimes DR(P_n)$

4 ENERGY ON TENSOR PRODUCT IN SOME STANDARD GRAPH

Definition 4.1

Let G be a simple graph with n vertices. Let A be the adjacency matrix of G, $\lambda_i, i = 1, 2, \cdots n$ be eigen value of A. The energy of the graph is defined as

$$E(G) = \sum_{i=1}^{n} |\lambda_i|$$

Definition 4.2

Let G be a simple graph with n vertices. Let A be the adjacency matrix of G, $\lambda_i, i = 1, 2, \cdots n$ be eigen value of A. The maximal energy of the graph is defined as

$$E_{\max}(G) = \sum_{i=1}^{n} \lceil |\lambda_i| \rceil$$

Definition 4.3

Let G be a simple graph with n vertices. Let A be the adjacency matrix of G, $\lambda_i, i = 1, 2, \cdots n$ be eigen value of A. The minimal energy of the graph is defined as

$$E_{\min}(G) = \sum_{i=1}^{n} \lfloor |\lambda_i| \rfloor$$

Theorem 4.4

Let P_n be odd path on n vertices. Then

(i) $E_{\max}[DR(P_2 \otimes P_n)] = 2(n+1)$

(ii) $E_{\min}[DR(P_2 \otimes P_n)] = 2(n-1)$

By using definition 4.2 and 4.3, we get

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Proof.

Let n is an odd path then,

$$A(DR(P_2 \otimes P_n)) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & & & \vdots & & & \vdots & & \vdots & & \ddots & \vdots & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

$$Ch(A(DR(P_2 \otimes P_n)), \lambda) = \begin{bmatrix} -\lambda & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 1 & -\lambda & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 1 & -\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \\ 0 & 0 & 0 & -\lambda & 1 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 1 & -\lambda & 1 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\lambda & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \\ 0 & 0 & 0 & 0 & 0 & 0 & -\lambda & 1 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\lambda & 0 & 0 & \cdots & 0 & 0 \\ \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda & 1 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\lambda & \cdots & 0 & 0 \\ \vdots & & & \vdots & & & \vdots & & \vdots & & \ddots & \vdots & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & -\lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & -\lambda \end{bmatrix}$$

The maximal energy of tensor product on P_2 and P_n is 2(n+1) and the minimal energy of tensor product on P_2 and P_n is 2(n-1).

i.e., $E_{\max}[DR(P_2 \otimes P_n)] = 2(n+1)$ and $E_{\min}[DR(P_2 \otimes P_n)] = 2(n-1)$

Theorem 4.5

Let P_n be even path on n vertices. Then $E\ [DR(P_2 \otimes P_n)] = 2n$

Proof.

Let n is an even path then,

$$A(DR(P_2 \otimes P_n)) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \\ 0 & 0 & 0 & 1 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \\ 0 & 0 & 0 & 0 & 0 & 1 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ & \vdots & & \vdots & & \vdots & & & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

$$Ch(A(DR(P_2 \otimes P_n)), \lambda) = \begin{bmatrix} -\lambda & 1 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 1 & -\lambda & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \\ 0 & 0 & -\lambda & 1 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 1 & -\lambda & 0 & 0 & \cdots & 0 & 0 \\ \\ 0 & 0 & 0 & 0 & -\lambda & 1 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\lambda & \cdots & 0 & 0 \\ & \vdots & & \vdots & & \vdots & & & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & & -\lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & -\lambda \end{bmatrix}$$

The characteristic polynomial is $(\lambda - 1)^n (\lambda + 1)^n = 0$

Hence, the energy of tensor product on P_2 and P_n is 2n.

Theorem 4.6

Let C_n be cycle of even length then energy of Tensor Product with Detour radial of P_2 and Detour radial of C_n is $2^{\frac{n}{2}}$.

Proposition 4.7

Let G be a simple graph and $\lambda_i, i = 1, 2, \cdots n$ be eigen value of the adjacent matrix A then the relation between energy, maximal energy and minimal energy is $E_{\min}(G) \le E(G) \le E_{\max}(G)$

Proof.

The proof is followed by the definition of energy, maximal energy and minimal energy.

Proposition 4.8

Let G be a simple graph and $\lambda_i, i = 1, 2, \cdots n$ be eigen value of the adjacent matrix A then $E_{\min}(G) < E_{\max}(G)$

Proof.

Since by preposition 4.7 and by the definition, this gives the direct result.

CONCLUSION

Thus in this paper, we find the b-Radial graph and Tensor Product in Detour Radial Graph for some standard graphs. Also maximal energy and minimal energy are defined and its exact values are obtained for Tensor Product in some standard graph.

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